Project IV 2025/26

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PROBABILITY ON TREES

Trees are a well-behaved type of graph, even when they are random; they do not have loops or cycles. Furthermore, when a specific vertex, known as the root, is designated, the tree naturally acquires a genealogical structure, with the root acting as the common ancestor of all the vertices in the tree. Additionally, each vertex has exactly one parent, which gives trees a recursive structure. This property is particularly useful when considering random or deterministic processes on trees, as it makes their study more manageable compared to other types of graphs.

the root v the descendants of v

The goal of this project is to study random trees and exploit their recursive nature by analyzing different processes on them, ranging from electrical networks to random walks, and from Bernoulli percolation to the Ising model. These models become significantly more manageable when studied on trees.

By the end of the first semester, we will have learned about Galton-Watson trees – a specific type of branching process – along with their properties, and we will have studied a general form of tree recursion within these trees. The objective for the second semester is to focus on a specific model on a Galton-Watson tree.

We will use "Probability on Trees and Networks" by Russell Lyons and Yuval Peres as our main reference, available on the first author's webpage.

This project allows for nice computer simulations, and many beautiful drawings.

Recommended prior knowledge:

- Probability II
- At least one of Stochastic Processes III or Markov Chains II

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